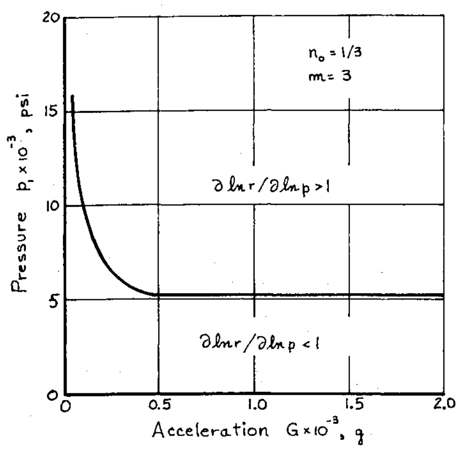

Fig. 2 Stability boundary for propellant P411.

tions. This is impossible within the present structure of the Sturm-Reichenbach theory. However, with the modification in the function f introduced herein this can be achieved if $m = \infty$. Although this guarantees that $(\partial \ln r/\partial \ln p)_\infty = n_0$, $\partial \ln r/\partial \ln p$ will still exceed n_0 for intermediate accelerations. Figure 3 illustrates $\partial \ln r/\partial \ln p$ for propellant P411 with $m = 3$ and $m = \infty$. The figure shows that with $m = \infty$ $\partial \ln r/\partial \ln p$ still attains a maximum value substantially greater than n_0. Note that for accelerations below $200g$, the dependence on m essentially vanishes.

The implications concerning $\partial \ln r/\partial \ln p$ stem from both the criteria determining d_{pc} and the assumption relating to the function f. The former appears to be on a sound physical basis. However, the latter is justified by recourse to null field theories. Since in an acceleration field this effect is determined by phenomena occurring between the "free" AP particles and the propellant, a phenomenon that does not occur in either null field theory, this justification seems weak. It is interesting to note that Willoughby and Crowe[4] abandoned the approach taken by Ref. 1 for one dealing directly with energy transport across the particle/surface clearance in order to achieve qualitative agreement regarding pressure dependence. According to the Phalanx flame model, burning rate is controlled by reaction at the AP/binder interface. The mechanism by which energy released by freed AP particles (this means they have been by-passed by the Phalanx flame) increases the Phalanx flame rate is obscure. Moreover, if the mass burning rate were to exceed the Phalanx rate, free AP particles would no longer be created. Therefore, the Phalanx rate appears to represent an upper bound on burning rate that is not included in the model as stated. Finally, data for TPG-3016D propellant (Thiokol designation) presented by Northam[5] show that $r/r_0 \neq 1$ when the acceleration is away from the burning surface—a phenomenon disallowed by the model. However, in Ref. 6, results do show $r/r_0 = 1$ with parallel accelerations.

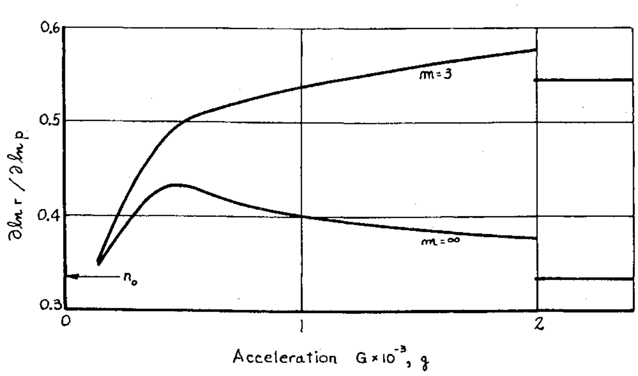

Fig. 3 $\partial \ln r/\partial \ln p$ at 1000 psi.

In summary the work reported here shows that the Sturm-Reichenbach theory leads to $\partial \ln r/\partial \ln p > n_0$ for $G > 0$. However, some data reported in the literature indicate that $\partial \ln r/\partial \ln p \simeq n_0$. In addition, some data on the variation of burning rate with acceleration direction conflicts with the theory as stated. These results suggest that the theory has limited validity. However, since the pressure dependency difficulties can be partially overcome and the theory provides a framework for interpolation/extrapolation with limited data, additional tests of this theory should be encouraged, especially those at high pressure (≥2000 psi) and high acceleration (≥200g).

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Reply by Authors to R. L. Glick

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Glick has derived some interesting consequences of the theory relating to the pressure and acceleration dependence of $\partial \ln r/\partial \ln p$. In deriving his Eqs. (4, 5, and 6) (from which subsequently were plotted the straight line portions of Fig. 1, and Figs. 2 and 3) Glick made use of the assumption that

$$\lim_{G \to \infty} J' d_{pc} = 0$$

Although this limit is mathematically correct, a zero value of $J'd_{pc}$ is not admissible within the framework of the model as it is postulated. As a result, Glick's Eqs. (4–6) are not considered valid. However, Eq. (2) and its consequences are worthy of note and will be discussed subsequent to the discussion of the term $J'd_{pc}$ and its limiting value from a physical standpoint.

Let us first consider the factor d_{pc}. Although not explicitly stated in the mathematical derivation, there exists on physical grounds a lower limit (greater than zero) for d_{pc}.

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Table 1 $\partial \ln r/\partial \ln p$ for P4111 with $n_0 = 0.208$

Acceleration	$\partial \ln r/\partial \ln p$ (500 psia)	$\partial \ln r/\partial \ln p$ (1000 psia)
50	...	0.380
100	0.350	...
250	...	0.427
500	0.372	0.440
1000	0.380	0.454
1500	0.380	...

The basis of the model are the assumptions that: 1) small ammonium perchlorate (AP) oxidizer particles evolved at the burning surface become separated from the fuel matrix by a thin gas layer when the Phalanx flame is able to burn completely around the AP particle-fuel matrix before the AP particles are consumed; and 2) the propellant burning rate increase in an acceleration field is proportional to the amount of additional energy that is transferred to the propellant surface as a result of the retention and combustion of these particles near the surface. The initial diameter of the smallest particle which is "freed" by the Phalanx flame and subsequently is retained on the surface and transfers additional energy to the propellant surface is called the critical diameter (d_{pc}). In keeping with the approximate nature of the model, it was assumed (invalidly) that none of the small AP particle mass was consumed either before or while being separated from the propellant surface by the Phalanx flame. This assumption was used to make the J vs d_{pc} dependence amenable to solution. Actually, the Phalanx flame will consume a portion of the oxidizer particle as it etches its way along the AP particle/fuel binder interface.

In view of this, one must realize, when considering a limiting value of d_{pc}, that the smallest freed AP particle, which is to remain on the surface and transfer some small but finite amount of energy to the surface, originally had a diameter d_{pc} greater than zero. Secondly, one must consider the size of the smallest oxidizer particle in the propellant when taking the limit of d_{pc} as the acceleration force approaches infinity. It is not physically meaningful to consider d_{pc} smaller than the smallest oxidizer particles contained in the propellant. Hence, from a physical standpoint, the limiting value of d_{pc} is the larger of either the diameter of the smallest oxidizer particle contained in the propellant or the initial diameter of an oxidizer particle which upon being for the most part consumed by the Phalanx flame is still able to transfer some small but finite amount of energy to the propellant surface. This diameter is clearly larger than zero.

For any given propellant there exists a definite relationship between J and d_{pc} for $d_{ps} \leq d_{pc} \leq d_{pl}$ where d_{ps} are the smallest and d_{pl} are the largest AP particles, respectively, in the small AP particle size distribution. It is physically meaningless to consider values of J and hence J' for $d_{pl} < d_{pc} < d_{ps}$. Hence for propellant P411 one cannot consider values of J' for $d_{pc} < 2.1\ \mu$.

The model, with its postulated mechanisms of AP particle separation and subsequent retention on the propellant surface, is applicable only for those values of acceleration and pressure, which when substituted into the equation $d_{pc} = K_1 (r/Gp)^{1/2}$, result in a d_{pc} no smaller than the physically limiting value discussed previously. Representative combinations of G and p for propellant P411, based on a d_{pc} no smaller than the smallest size AP particle contained in the propellant (2.1 μ) are $p = 500$ psia, $G = 1575$; $p = 1000$ psia, $G = 1055$; and $p = 5000$ psia, $G = 522$. These values encompass the values of general interest in rocket propulsion and are not considered to seriously limit the applicability of the model. To consider the model valid beyond these values of acceleration and pressure for P411 is to deny one or more of the postulated mechanisms on which the model is based.

Glick attributes the discontinuities in his Fig. 1 to the lack of smoothness in the J function at $d_{pc} = d_{ps}$, i.e. $J = 1$ and $J' = 0$ for $d_{pc} \leq d_{ps}$. One cannot assign meaningful values of the factors J' and d_{pc} beyond those allowed on physical grounds. Note that the discontinuities in Fig. 1 occur well beyond those G levels for which the model is considered applicable.

Equation (2) was used to calculate the values of $\partial \ln r/\partial \ln p$ for P411 with $n_0 = 0.208$ which was the experimentally determined value. The results are tabulated in Table 1. These results yield a very flat curve for values of acceleration beyond $100g$ as compared to the curves plotted by Glick. However, at low-g levels the value of n does increase fairly rapidly with increasing acceleration. It should be noted that all three of the Ref. 1‡ propellants did exhibit a rapidly increasing value of n at low-acceleration levels with acceleration as did Ref. 3 propellant ×101. On the other hand, the Ref. 3 propellant ×301 exhibited a decreasing value of n with acceleration. It appears that more experimental work should be done to establish a definitive trend of the acceleration dependence of the burning rate exponent n.

Glick found it interesting that Willoughby and Crowe abandoned the approach taken in Ref. 1. This is not considered relevant since Willoughby and Crowe were modeling metallized propellants which experience unsteady burning and surface "flooding" when subjected to acceleration forces.

The following explanations are offered to Glick's questions regarding the mechanisms of the Ref. 1 model and their implications on the phalanx flame model: 1) The freed AP particles increase the Phalanx flame rate to the extent that the final burned gas temperature region is nearer to the propellant surface, i.e., increased temperature gradients, because the body forces have retained these AP particles on the propellant surface. 2) The freed AP particles have not been by-passed by the Phalanx flame. To the contrary, they have been freed by the Phalanx flame before they were completely consumed. 3) It is agreed that the Phalanx rate represents an upper bound on the burning rate under any acceleration.

In summary, Glick has shown that the present model yields the interesting result that the burning rate exponent increases with acceleration. However, those conclusions which Glick bases on a zero limiting value of $J'd_{pc}$ are believed to be invalid. We believe that the model is limited in its applicability to finite values of acceleration and pressure as explained previously. The model is not at variance with the basic Phalanx flame model.

‡ All references are those specified in Glick's comments.